Algebraic renormalization of N = 2 Super Yang–Mills theories coupled to matter

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ABSTRACT

We study the algebraic renormalization of $N = 2$ Supersymmetric Yang–Mills theories coupled to matter. A regularization procedure preserving both the BRS invariance and the supersymmetry is not known yet, therefore it is necessary to adopt the algebraic method of renormalization, which does not rely on any regularization scheme. The whole analysis is reduced to the solution of cohomology problems arising from the generalized Slavnov operator which summarizes all the symmetries of the model. Besides to unphysical renormalizations of the quantum fields, we find that the only coupling constant of $N = 2$ SYMs can get quantum corrections. Moreover we prove that all the symmetries defining the theory are algebraically anomaly–free.

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1 Introduction

Since the early days of quantum field theory, physicists considered the property of finiteness as one of the most appealing features a theory could possess, according to the belief that the ultimate Theory of Nature should be finite [1]. In the framework of quantum field theory, the finiteness of a model results in the absence of quantum redefinitions of the physical parameters – and possibly of the quantum fields –, meaning that the corresponding β–functions – together with the anomalous dimensions related to the renormalizations of the quantum fields –, vanish, which is also equivalent to say that the theory does not exhibit ultraviolet divergences.

The supersymmetric field theories were the first to show a good ultraviolet behaviour [2], and actually this represented the original motivation for the interest arosen around them. In particular, the most spectacular renormalization properties occurr in Supersymmetric Yang–Mills theories (SYMs) with extended ($N \geq 2$) supersymmetry and calculations at higher and higher loops [3] strongly supported the conjecture that the maximally extended one, namely $N = 4$ SYM, was finite [4, 5]. Only recently a formal proof of its finiteness to all orders of perturbations theory has been given [6].

As a matter of fact the $N = 4$ case can be interpreted as a $N = 2$ SYM with matter in the adjoint representation of the gauge group and for this reason the gauge field theories with extended $N = 2$ supersymmetry are considered as the most general ones [7]. One important property is that not every $N = 2$ SYM is finite, but divergent quantum

corrections, if any, occurr only at one loop [4, 8]. Moreover, the physical relevance of $N = 2$ SYMs has been stressed in a couple of recent and authoritative papers [9], which made them popular and fashionable, thanks to some exact results concerning very important topics like confinement and electric–magnetic duality.

The study of the ultraviolet behaviour of a quantum field theory is included in the renormalization program, which more generally concerns the analysis of its divergence structure and the discussion of the possible extension to the quantum level of the symmetries characterizing it. When dealing with a supersymmetric theory, the absence of an acceptable regularization procedure preserving both the BRS invariance and the supersymmetry renders mandatory the adoption of the algebraic method of renormalization, which indeed does not rely on any underlying regularization scheme, and, moreover, leads to results valid to all orders of perturbation theory.

The first algebraic study of the renormalizability of a supersymmetric gauge field theory has been done for $N = 1$ SYM in the superfield formalism [10, 11], and recently the same results have been recovered within the components description [12]. Also the already cited proof of the finiteness of $N = 4$ SYM [6] has been given by using the algebraic method and adopting the Wess–Zumino gauge, the superfield formalism presenting more difficulties than advantages for the purpose of renormalizing field theories with extended supersymmetry. The complete algebraic renormalization of the general $N = 2$ SYM is still lacking, the only attempt in that direction being still uncompleted [13]. The difficulties encountered in [13] originated from the infinite dimensional algebraic structure, controlled

at the price – which resulted too high for the renormalization of the model – of introducing an infinite number of external sources with increasing negative dimension.

Inspired by the algebraic proofs of the finiteness of the topological quantum field theories [14], which present analogous supersymmetric algebraic structures, we gave in [15] a formulation of $N = 2$ SYMs alternative to that presented in [13], having the advantage of being characterized by an algebra closed without making use neither of equations of motion nor of auxiliary fields. The role of the latter is indeed played by the external sources coupled to the nonlinear variations of the quantum fields, and therefore necessarily present in the theory [16, 17]. The essence of the method followed in [15] was to collect all the symmetries defining the theory into one generalized Slavnov operator, so that the once complicated algebra reduced to a simple nilpotency relation. In this paper we give the quantum extension of the classical discussion made in [15], which be briefly summarize in Section 2. In Section 3 we perform the renormalization of the model, which formally is that of an ordinary gauge field theory described by a Slavnov identity : first, we study the stability of the classical action under radiative corrections and then we seek for possible anomalies. Some conclusions are drawn in Section 4.

2 The classical model

In this section we briefly review the classical properties of the theory which are necessary for what follows, referring to [15] for more details. The fields of $N = 2$ SYM are organized according to the vector multiplet $(A^a_\mu, \lambda_{\alpha i}, A^a, B^a)$, belonging to the adjoint representation of a gauge group G, and the matter multiplet $(A^{iA}, A^*_{iA}, \psi^\alpha_A, \bar{\psi}^A_{\dot\alpha})$, which is in an arbitrary representation. In addition to these physical fields, a ghost c^a, an antighost $\bar{c}^a$ and a Lagrange multiplier b^a are introduced according to the usual gauge–fixing procedure. The $N = 2$ SYM is described by the complete gauge–fixed classical action

$$\Sigma \equiv S_{inv} + S_{gf} + S_{ext} \ , \tag{2.1}$$

where

$$\begin{aligned}
S_{inv} = \ & S_{SYM} + S_{matter} + S_{int} \\
= \ & \tfrac{1}{g^2} \int d^4x \Bigg(-\frac{1}{4} F^{a\mu\nu} F^a_{\mu\nu} + \frac{1}{2} (D^\mu A)^a (D_\mu A)^a + \frac{1}{2} (D^\mu B)^a (D_\mu B)^a \\
& + \tfrac{1}{2} \bar{\lambda}^{ai} \gamma^\mu (D_\mu \lambda_i)^a + \tfrac{1}{2} f^{abc} (\bar{\lambda}^{bi} \lambda^c_i) A^a - i \tfrac{1}{2} f^{abc} (\bar{\lambda}^{bi} \gamma_5 \lambda^c_i) B^a - \tfrac{1}{2} f^{amn} A^m B^n f^{apq} A^p B^q \Bigg) \\
& + \int d^4x \left((D^\mu A^i)^A (D_\mu A^*_i)_A - \frac{1}{2} \bar{\psi}_A \gamma^\mu (D_\mu \psi)^A \right) \\
& + \int d^4x \Bigg(- (T^a)^A_B (\bar{\psi}_A \lambda^a_i) A^{iB} + (T^a)^A_B (\bar{\lambda}^{ai} \psi^B) A^*_{iA} + (T^a)^A_D (T^b)^D_B A^a A^b A^*_{iA} A^{iB} \\
& + (T^a)^A_D (T^b)^D_B B^a B^b A^*_{iA} A^{iB} - i \tfrac{1}{2} (T^a)^A_B (\bar{\psi}_A \gamma_5 \psi^B) B^a - \tfrac{1}{2} (T^a)^A_B (\bar{\psi}_A \psi^B) A^a \Bigg) \ ,
\end{aligned} \tag{2.2}$$

$$S_{gf} = \int d^4x \left(b^a \partial A^a - (\partial^\mu \bar{c}^a)(D_\mu c)^a + (\partial^\mu \bar{c}^a)(\bar{\varepsilon}^i \gamma_\mu \lambda^a_i) \right) \ , \tag{2.3}$$

$$
\begin{aligned}
S_{ext} = \int d^4x \Bigg(\quad & M^a(\mathcal{Q}A^a) + N^a(\mathcal{Q}B^a) + \Omega^{a\mu}(\mathcal{Q}A^a_\mu) + \bar{\Lambda}^{ai}(\mathcal{Q}\lambda^a_i) + L^a(\mathcal{Q}c^a) \\
& + U^*_{iA}(\mathcal{Q}A^{iA}) + U^{iA}(\mathcal{Q}A^*_{iA}) + \bar{\Psi}_A(\mathcal{Q}\psi^A) + (\mathcal{Q}\bar{\psi}_A)\Psi^A \\
& - (\bar{\varepsilon}^j\Lambda^a_i)(\bar{\Lambda}^{ai}\varepsilon_j) + \tfrac{1}{2}(\bar{\varepsilon}^j\Lambda^a_j)(\bar{\Lambda}^{ai}\varepsilon_i) \\
& - (\bar{\varepsilon}^i\varepsilon_i)(\bar{\Psi}_A\Psi^A) + (\bar{\varepsilon}^i\gamma_5\varepsilon_i)(\bar{\Psi}_A\gamma_5\Psi^A) + (\bar{\varepsilon}^i\gamma^\mu\varepsilon_i)(\bar{\Psi}_A\gamma_\mu\Psi^A) \Bigg) \ .
\end{aligned} \tag{2.4}
$$

Notice the absence of a bilinear term in the Lagrange multiplier b in the gauge–fixing part S_{gf} of the action, which corresponds to choosing the Landau gauge.

In (2.1), $S_{inv} + S_{gf}$ is left invariant by the operator $\mathcal{Q}$ which sums up BRS, supersymmetry and translations by means of two ghost charged parameters ε_i and ξ^μ :

$$
\begin{aligned}
\mathcal{Q}A^a = \;& f^{abc}c^bA^c + \bar{\varepsilon}^i\lambda^a_i + \xi^\mu\partial_\mu A^a \\
\mathcal{Q}B^a = \;& f^{abc}c^bB^c + i\bar{\varepsilon}^i\gamma_5\lambda^a_i + \xi^\mu\partial_\mu B^a \\
\mathcal{Q}A^a_\mu = \;& -(D_\mu c)^a + \bar{\varepsilon}^i\gamma_\mu\lambda^a_i + \xi^\nu\partial_\nu A^a_\mu \\
\mathcal{Q}\lambda^a_{\alpha i} = \;& f^{abc}c^b\lambda^c_{\alpha i} + \tfrac{1}{2}F^a_{\mu\nu}(\sigma^{\mu\nu}\varepsilon_i)_\alpha - (D_\mu A)^a(\gamma^\mu\varepsilon_i)_\alpha \\
& + i(D_\mu B)^a(\gamma^\mu\gamma_5\varepsilon_i)_\alpha + if^{abc}A^bB^c(\gamma_5\varepsilon_i)_\alpha + \xi^\mu\partial_\mu\lambda^a_{\alpha i} \\
\mathcal{Q}A^{iA} = \;& (T^a)^A_Bc^aA^{iB} + \bar{\varepsilon}^i\psi^A + \xi^\mu\partial_\mu A^{iA} \\
\mathcal{Q}\psi^A_\alpha = \;& (T^a)^A_Bc^a\psi^B_\alpha - 2(D_\mu A^i)^A(\gamma^\mu\varepsilon_i)_\alpha + 2(T^a)^A_BA^{iB}A^a\varepsilon_{\alpha i} \\
& - 2i(T^a)^A_BA^{iB}B^a(\gamma_5\varepsilon_i)_\alpha + \xi^\mu\partial_\mu\psi^A_\alpha \\
\mathcal{Q}c^a = \;& \tfrac{1}{2}f^{abc}c^bc^c - (\bar{\varepsilon}^i\gamma^\mu\varepsilon_i)A^a_\mu - i(\bar{\varepsilon}^i\gamma_5\varepsilon_i)B^a + (\bar{\varepsilon}^i\varepsilon_i)A^a + \xi^\mu\partial_\mu c^a \\
\mathcal{Q}\bar{c}^a = \;& b^a + \xi^\mu\partial_\mu\bar{c}^a \\
\mathcal{Q}b^a = \;& (\bar{\varepsilon}^i\gamma^\mu\varepsilon_i)\partial_\mu\bar{c}^a + \xi^\mu\partial_\mu b^a \\
\mathcal{Q}\xi^\mu = \;& -(\bar{\varepsilon}^i\gamma^\mu\varepsilon_i) \\
\mathcal{Q}\varepsilon_i = \;& 0 \ .
\end{aligned} \tag{2.5}
$$

The operator $\mathcal{Q}$ is nilpotent provided that the spinor field equations are satisfied

$$\mathcal{Q}^2 = \text{equations of motion} \ , \tag{2.6}$$

and this implies the presence in S_{ext} of terms quadratic in the external sources, in addition to the usual couplings to the nonlinear $\mathcal{Q}$–transformations of the quantum fields. With such a source term in the total classical action Σ it is possible to write the generalized Slavnov identity

$$\begin{aligned}
\mathcal{S}(\Sigma) = \quad & \int d^4x \left(\frac{\delta\Sigma}{\delta\Omega^{a\mu}}\frac{\delta\Sigma}{\delta A^a_\mu} + \frac{\delta\Sigma}{\delta L^a}\frac{\delta\Sigma}{\delta c^a} + \frac{\delta\Sigma}{\delta M^a}\frac{\delta\Sigma}{\delta A^a} + \frac{\delta\Sigma}{\delta N^a}\frac{\delta\Sigma}{\delta B^a} + \frac{\delta\Sigma}{\delta\bar\Lambda^{ai}}\frac{\delta\Sigma}{\delta\lambda^a_i} \right. \\
& + \frac{\delta\Sigma}{\delta U^*_{iA}}\frac{\delta\Sigma}{\delta A^{iA}} + \frac{\delta\Sigma}{\delta U^{iA}}\frac{\delta\Sigma}{\delta A^*_{iA}} + \frac{\delta\Sigma}{\delta\bar\Psi_A}\frac{\delta\Sigma}{\delta\psi^A} + \frac{\delta\Sigma}{\delta\Psi^A}\frac{\delta\Sigma}{\delta\bar\psi_A} \\
& \left. + (b^a + \xi^\mu\partial_\mu\bar c^a)\frac{\delta\Sigma}{\delta\bar c^a} + ((\bar\varepsilon^i\gamma^\mu\varepsilon_i)\partial_\mu\bar c^a + \xi^\mu\partial_\mu b^a)\frac{\delta\Sigma}{\delta b^a} \right) - (\bar\varepsilon^i\gamma^\mu\varepsilon_i)\frac{\partial\Sigma}{\partial\xi^\mu} \\
= \quad & 0 \ .
\end{aligned} \tag{2.7}$$

The corresponding linearized Slavnov operator

$$\begin{aligned}
\mathcal{B}_\Sigma = \int d^4x & \left(\frac{\delta\Sigma}{\delta\Omega^{a\mu}}\frac{\delta}{\delta A^a_\mu} + \frac{\delta\Sigma}{\delta A^a_\mu}\frac{\delta}{\delta\Omega^{a\mu}} + \frac{\delta\Sigma}{\delta L^a}\frac{\delta}{\delta c^a} + \frac{\delta\Sigma}{\delta c^a}\frac{\delta}{\delta L^a} + \frac{\delta\Sigma}{\delta M^a}\frac{\delta}{\delta A^a} + \frac{\delta\Sigma}{\delta A^a}\frac{\delta}{\delta M^a} \right. \\
& + \frac{\delta\Sigma}{\delta N^a}\frac{\delta}{\delta B^a} + \frac{\delta\Sigma}{\delta B^a}\frac{\delta}{\delta N^a} + \frac{\delta\Sigma}{\delta\bar\Lambda^{ai}}\frac{\delta}{\delta\lambda^a_i} - \frac{\delta\Sigma}{\delta\lambda^a_i}\frac{\delta}{\delta\bar\Lambda^{ai}} + \frac{\delta\Sigma}{\delta U^*_{iA}}\frac{\delta}{\delta A^{iA}} + \frac{\delta\Sigma}{\delta A^{iA}}\frac{\delta}{\delta U^*_{iA}} \\
& + \frac{\delta\Sigma}{\delta U^{iA}}\frac{\delta}{\delta A^*_{iA}} + \frac{\delta\Sigma}{\delta A^*_{iA}}\frac{\delta}{\delta U^{iA}} + \frac{\delta\Sigma}{\delta\bar\Psi_A}\frac{\delta}{\delta\psi^A} - \frac{\delta\Sigma}{\delta\psi^A}\frac{\delta}{\delta\bar\Psi_A} + \frac{\delta\Sigma}{\delta\Psi^A}\frac{\delta}{\delta\bar\psi_A} - \frac{\delta\Sigma}{\delta\bar\psi_A}\frac{\delta}{\delta\Psi^A} \\
& \left. + (b^a + \xi^\mu\partial_\mu\bar c^a)\frac{\delta}{\delta\bar c^a} + ((\bar\varepsilon^i\gamma^\mu\varepsilon_i)\partial_\mu\bar c^a + \xi^\mu\partial_\mu b^a)\frac{\delta}{\delta b^a} \right) - (\bar\varepsilon^i\gamma^\mu\varepsilon_i)\frac{\partial}{\partial\xi^\mu}
\end{aligned} \tag{2.8}$$

as a consequence of (2.7) is off–shell nilpotent

$$\mathcal{B}_\Sigma\mathcal{B}_\Sigma = 0 \ . \tag{2.9}$$

Finally, in addition to the Slavnov identity (2.7), the classical theory is defined by three further constraints :

1. the gauge condition

$$\frac{\delta\Sigma}{\delta b^a} = \partial A^a \ , \tag{2.10}$$

whose commutator with the Slavnov identity (2.7) gives the antighost equation

$$\bar{\mathcal{F}}^a\Sigma \equiv \frac{\delta\Sigma}{\delta\bar{c}^a} + \partial^\mu\frac{\delta\Sigma}{\delta\Omega^{a\mu}} - \xi^\mu\partial_\mu\frac{\delta\Sigma}{\delta b^a} = 0 \ ; \tag{2.11}$$

2. the ξ–equation

$$\frac{\partial\Sigma}{\partial\xi^\mu} = \Delta_\mu \ , \tag{2.12}$$

where

$$\begin{aligned}\Delta_\mu \equiv \textstyle\int d^4x \Big(& -M^a\partial_\mu A^a - N^a\partial_\mu B^a - \Omega^{a\nu}\partial_\mu A^a_\nu - (\bar{\Lambda}^{ai}\partial_\mu\lambda^a_i) + L^a\partial_\mu c^a \\ & -U^*_{iA}\partial_\mu A^{iA} - U^{iA}\partial_\mu A^*_{iA} - (\bar{\Psi}_A\partial_\mu\psi^A) + (\partial_\mu\bar{\psi}_A\Psi^A)\Big) \ ;\end{aligned} \tag{2.13}$$

3. the ghost equation of the Landau gauge [18, 17]

$$\mathcal{F}^a\Sigma = \Delta^a \ , \tag{2.14}$$

where

$$\mathcal{F}^a \equiv \int d^4x \left(\frac{\delta}{\delta c^a} + f^{abc}\bar{c}^b\frac{\delta}{\delta b^c}\right) \tag{2.15}$$

and

$$
\begin{aligned}
\Delta^a \equiv \int d^4x \Bigg(\; & f^{abc}\Big(M^b A^c + N^b B^c + \Omega^{b\mu} A^c_\mu + (\bar{\Lambda}^{bi}\lambda^c_i) - L^b c^c\Big) \\
& -(T^a)^A_B\Big(U^*_{iA} A^{iB} - U^{iB} A^*_{iA} + (\bar{\Psi}_A \psi^B) + (\bar{\psi}_A \Psi^B)\Big)\Bigg) \; .
\end{aligned}
\tag{2.16}
$$

3 Renormalization

The proof of the renormalizability of the theory consists in showing that it is possible to define a quantum vertex functional

$$
\Gamma = \Sigma + O(\hbar) \tag{3.1}
$$

which coincides at the lowest perturbative order with the classical action Σ (2.1), and which satisfies the generalized Slavnov identity

$$
\mathcal{S}(\Gamma) = 0 \; . \tag{3.2}
$$

The algebraic renormalization scheme is performed according to two independent steps. First we shall study the *stability* of the classical action Σ under radiative corrections, checking that the most general invariant counterterm can be reabsorbed through a redefinition of the fields and of the only coupling constant g^2 of the theory. Then we shall discuss the presence of *anomalies*, namely we shall investigate wether the symmetries

defining the theory can be implemented at the quantum level.

This problem was addressed by Breitenlohner and Maison in [13], but they encountered some difficulties originating from the algebraic structure of $N = 2$ SYMs. The supersymmetry algebra finds two obstructions to the closure on the translations: equations of motion and field dependent gauge transformations [15]. Following a standard procedure, the authors of [13] introduced auxiliary fields in order to eliminate the equations of motion. Still, the presence of the field dependent gauge transformations kept the algebra infinite dimensional and therefore an infinite number of external sources with increasing negative dimension were needed in order to control the supersymmetric structure. This rendered the analysis of the renormalization quite difficult and consequently a complete discussion of the renormalization of $N = 2$ SYM was never achieved.

The approach we are following here is different because the classical Slavnov identity (2.7) has been obtained in [15] by collecting into an unique operator all the symmetries of the theory. Consequently, its quantum extension corresponds to that of all the symmetries partecipating in it, in particular the BRS transformations and the $N = 2$ supersymmetry. In other words, the absence of anomalies for the Slavnov identity (2.7) implies that both the BRS symmetry and the N=2 supersymmetry are anomaly–free as well [19]. Moreover, the study of the stability of the classical action and of the anomalies technically reduces to the analysis of cohomologies of the linearized Slavnov operator [6, 12, 14, 20], which is a far easier task than that encountered in [13].

3.1 Stability of the classical action

In order to find the most general invariant local counterterm, we perturb the classical action

$$\Sigma \longrightarrow \Sigma + \eta \Sigma_c \ , \tag{3.3}$$

where η is an infinitesimal parameter and Σ_c is the most general integrated local functional with canonical dimension four and Faddeev–Popov ($\Phi\Pi$) charge zero. We then require that the perturbed action satisfies the symmetries defining the theory. At first order in η this corresponds to imposing the following constraints on the functional Σ_c :

1. the gauge condition

$$\frac{\delta}{\delta b^a}(\Sigma + \eta\Sigma_c) = \partial A^a \quad \Rightarrow \quad \frac{\delta \Sigma_c}{\delta b^a} = 0 \ ; \tag{3.4}$$

2. the antighost equation

$$\bar{\mathcal{F}}^a(\Sigma + \eta\Sigma_c) = 0 \quad \Rightarrow \quad \frac{\delta \Sigma_c}{\delta \bar{c}^a} + \partial^\mu \frac{\delta \Sigma_c}{\delta \Omega^{a\mu}} = 0 \ ; \tag{3.5}$$

3. the ghost equation

$$\mathcal{F}^a(\Sigma + \eta\Sigma_c) = \Delta^a \quad \Rightarrow \quad \int d^4x \, \frac{\delta \Sigma_c}{\delta c^a} = 0 \ , \tag{3.6}$$

   where $\mathcal{F}^a$ and Δ^a are given by (2.15) and (2.16) respectively;

4. the ξ–equation

$$\frac{\partial}{\partial \xi^\mu}(\Sigma + \eta\Sigma_c) = \Delta_\mu \quad \Rightarrow \quad \frac{\partial \Sigma_c}{\partial \xi^\mu} = 0 \ , \tag{3.7}$$

where Δ_μ is given by (2.13);

5. the Slavnov identity (2.7), which at first order in η implies the invariance of the perturbation Σ_c under the action of the linearized Slavnov operator (2.8)

$$\mathcal{S}(\Sigma + \eta\Sigma_c) = 0 \quad \Rightarrow \quad \mathcal{B}_\Sigma \Sigma_c = 0 \ . \tag{3.8}$$

The conditions (3.4) and (3.7) are satisfied by a functional which does not depend neither on the Lagrange multiplier b^a nor on the global ghost ξ^μ. The antighost equation (3.5) implies that the external source $\Omega^{a\mu}$ and the antighost $\bar{c}^a$ appear in Σ_c only through the combination

$$\eta^{a\mu} \equiv \partial^\mu \bar{c}^a + \Omega^{a\mu} \ , \tag{3.9}$$

while the effect of the ghost equation (3.6) is that the perturbation depends on the ghost field c^a only if differentiated ($\partial_\mu c \equiv c_\mu$). A functional satisfying all previous constraints depends on the fields and parameters listed in Table 1 together with their quantum numbers.

	A^a_μ	$\lambda^a_{\alpha i}$	A^a	B^a	A^{iA}	ψ^α_A	c^a_μ	$\eta^{a\mu}$	$\Lambda_{\alpha i}$	M^a	N^a	U^{iA}	Ψ^A	L^a	ε_i
dim	1	3/2	1	1	1	3/2	1	3	5/2	3	3	3	5/2	4	−1/2
ΦΠ	0	0	0	0	0	0	1	1	1	1	1	1	1	2	1

Table 1. Dimensions and Faddeev–Popov charges

In the appendix we give the explicit form of the most general perturbation Σ_c, by classifying it according to eigenstates of the counting operator $\mathcal{N}_\varepsilon \equiv \bar{\varepsilon}^i \dfrac{\partial}{\partial \bar{\varepsilon}^i}$

$$\Sigma_c = \Sigma_c^{(0)} + \Sigma_c^{(1)} + \Sigma_c^{(2)} \ . \tag{3.10}$$

The counterterm Σ_c must satisfy the Slavnov condition (3.8), which constitutes a cohomology problem, due to the nilpotency of the linearized Slavnov operator $\mathcal{B}_\Sigma$ (2.8). The general solution of eq. (3.8) is

$$\Sigma_c = \Sigma_c^{(ph)} + \mathcal{B}_\Sigma \widehat{\Sigma}_c \ . \tag{3.11}$$

A necessary condition for the renormalizability of the theory is that the whole counterterm Σ_c can be reabsorbed by a redefinition of the quantum fields and of the coupling constant g^2 of the classical action. In particular, $\widehat{\Sigma}_c$ corresponds to unphysical field renormalizations, called anomalous dimensions, while $\Sigma_c^{(ph)}$, which cannot be written as a $\mathcal{B}_\Sigma$–variation, entails a nonvanishing β–function of the coupling constant g^2. Precisely, the physical renormalizations belong to the cohomology sector with vanishing ΦΠ–charge of the linearized Slavnov operator.

A long and straightforward calculation yields the following result for the Slavnov condition (3.8)

$$
\begin{aligned}
\Sigma_c = \quad & Z_{g^2}\Bigg(S_{inv} + \int d^4x \Big(-(\bar{\varepsilon}^j\Lambda^a_i)(\bar{\Lambda}^{ai}\varepsilon_j) + \frac{1}{2}(\bar{\varepsilon}^j\Lambda^a_j)(\bar{\Lambda}^{ai}\varepsilon_i) - (\bar{\varepsilon}^i\varepsilon_i)(\bar{\Psi}_A\Psi^A) \\
& +(\bar{\varepsilon}^i\gamma_5\varepsilon_i)(\bar{\Psi}_A\gamma_5\Psi^A) + (\bar{\varepsilon}^i\gamma^\mu\varepsilon_i)(\bar{\Psi}_A\gamma_\mu\Psi^A)\Big)\Bigg) \\
& +\mathcal{B}_\Sigma \int d^4x \Big(c_1\eta^{a\mu}A^a_\mu + c_2M^aA^a + c_3N^aB^a + c_4\bar{\Lambda}^{ai}\lambda^a_i \\
& +c_5(U^*_{iA}A^{ia} + U^{iA}A^*_{iA}) + c_6(\Psi_A\psi^A + \bar{\psi}_A\Psi^A)\Big) \ ,
\end{aligned}
\tag{3.12}
$$

where S_{inv} is the classical invariant action (2.2), while Z_{g^2} and c_i are constants related to the renormalization of g^2 and of the fields respectively.

The celebrated finiteness property of the supersymmetric theories translates either into the vanishing of the whole counterterm due to the algebraic conditions on it – as it happens for the topological models [14], or into the lack of the physical part of it $\Sigma_c^{(ph)}$ [21]. Here on the contrary we find that $N = 2$ SYMs exhibit a possible renormalization of the coupling constant, besides to anomalous dimensions for the quantum fields belonging to the vector and matter multiplet. Notice also that, as in ordinary gauge field theories built in the Landau gauge [22], the ghost field c does not renormalize because of the ghost equation (3.6) [18, 17].

The result (3.12) is the best one can obtain with the algebraic method of renormalization, according to which any claim on the coefficients appearing in the counterterm must be supported by a non anomalous symmetry of the classical action. On the other hand, within the superspace background field formulation of extended supersymmetry, it has

been possible to show that the $N = 2$ SYMs, whose $\beta(g)$ function is vanishing at one loop, are finite to all orders of perturbation theory [8], and an analogous nonrenormalization theorem has been proved for $N = 1$ SYM [21] by exploiting the fact that the R–current, the supersymmetry current and the energy momentum tensor are components of one superfield. Within the $N = 2$ SYMs, a particular role is played by the maximally extended $N = 4$ case, which can be interpreted as a $N = 2$ theory with matter in the adjoint representation of the gauge group. In that case, indeed, it has been possible to prove algebraically the perturbative finiteness, by showing the absence of the superconformal anomaly [6]

3.2 Anomalies

The symmetries characterizing the theory are acceptable for any purpose, including for instance the determination of the counterterm, only if they survive the process of quantization. In our framework, this entails the possibility of writing the quantum Slavnov identity (3.2). The standard algebraic procedure in order to prove that quantum implementation is to assume that the Slavnov identity gets broken at the quantum level by an insertion

$$\mathcal{S}(\Gamma) = \Delta \cdot \Gamma \ . \tag{3.13}$$

A fundamental information on the breaking is provided by the Quantum Action Principle [23], which states that at the lowest nonvanishing order in $\hbar$ the insertion $\Delta \cdot \Gamma$ is an

integrated local functional Δ with dimension four and $\Phi\Pi$–charge one

$$\Delta \cdot \Gamma = \Delta + O(\hbar\Delta) \ . \tag{3.14}$$

It is easy to show the validity to all orders of perturbation theory of the gauge condition (2.10), the antighost equation (2.11), the ξ–equation (2.12) and the ghost equation (2.14) [24]

$$\begin{array}{ll} \dfrac{\delta\Gamma}{\delta b^a} = \partial A^a & \bar{\mathcal{F}}^a\Gamma = 0 \\ \dfrac{\partial\Gamma}{\partial\xi^\mu} = \Delta_\mu & \mathcal{F}^a\Gamma = \Delta^a \ . \end{array} \tag{3.15}$$

The following algebraic relations hold

$$\mathcal{B}_\gamma \mathcal{S}(\gamma) = 0 \tag{3.16}$$

$$\begin{aligned} \frac{\delta}{\delta b^a}\mathcal{S}(\gamma) - \mathcal{B}_\gamma\Big(\frac{\delta\gamma}{\delta b^a} - \partial A^a\Big) = \ & \bar{\mathcal{F}}^a\gamma \\ \bar{\mathcal{F}}^a\mathcal{S}(\gamma) + \mathcal{B}_\gamma\bar{\mathcal{F}}^a\gamma = \ & 0 \\ \frac{\partial}{\partial\xi^\mu}\mathcal{S}(\gamma) + \mathcal{B}_\gamma\Big(\frac{\partial\gamma}{\partial\xi^\mu} - \Delta_\mu\Big) = \ & \mathcal{P}_\mu\gamma \\ \mathcal{F}^a\mathcal{S}(\gamma) + \mathcal{B}_\gamma(\mathcal{F}^a\gamma - \Delta^a) = \ & \mathcal{H}^a_{rig}\gamma \end{aligned} \tag{3.17}$$

where γ is a generic functional, $\mathcal{P}_\mu$ and $\mathcal{H}^a_{rig}$ are the Ward operator for translations and rigid gauge invariance respectively. Substituting in (3.16) and (3.17) the generic functional γ with the quantum vertex functional Γ satisfying the relations (3.15) and assuming as valid to all orders

$$\mathcal{P}_\mu\Gamma = \mathcal{H}^a_{rig}\Gamma = 0 \ , \tag{3.18}$$

the algebra (3.17) yields the following constraints on the lowest order breaking of the quantum Slavnov identity

$$\frac{\delta\Delta}{\delta b^a} = \bar{\mathcal{F}}^a\Delta = \frac{\partial\Delta}{\partial\xi^\mu} = \mathcal{F}^a\Delta = 0 \ , \tag{3.19}$$

which are satisfied by a functional depending only on the fields and parameters listed in Table 1. In addition to the constraints (3.19), the breaking Δ is subjected to the Wess–Zumino consistency condition [25] arising from (3.16)

$$\mathcal{B}_\Sigma\Delta = 0 \ . \tag{3.20}$$

The equation (3.20) is a cohomology problem like the Slavnov condition (3.8) for the stability of the theory. The difference is that this time the solution must belong to the space of local integrated functionals with canonical dimension four and $\Phi\Pi$–charge *one* instead of zero. The most general functional obeying the Wess–Zumino condition is

$$\Delta = \mathcal{A} + \mathcal{B}_\Sigma\widehat{\Delta} \ , \tag{3.21}$$

$\mathcal{A}$ being a closed and not exact form

$$\mathcal{A} \neq \mathcal{B}_\Sigma\widehat{\mathcal{A}} \ . \tag{3.22}$$

If $\mathcal{A}$ is present, or, equivalently, if the cohomology of the linearized Slavnov operator $\mathcal{B}_\Sigma$ in the space of the solutions of equation (3.20) is not empty, the breaking Δ cannot be

reabsorbed by a fine tuning of the fields and parameters of Γ. The functional $\mathcal{A}$ is an anomaly, namely an obstruction to the validity of the quantum Slavnov identity (3.2). On the contrary, if there is no anomaly ($\mathcal{A} = 0$), the equations (3.20) and (3.21) imply that the Slavnov identity holds good at a fixed order in $\hbar$ and hence, by induction, at every order.

The equation (3.20) can be solved using the method of spectral sequences [19] or by looking directly for its general solution. Either way, the calculation, although quite laborious, *per se* does not present particular difficulties. We solved the cohomology problem (3.20) by writing the candidate for the anomaly as the most general integrated local functional with dimension four, ΦΠ–charge one and depending on the fields listed in Table 1. The resulting functional Δ is the sum of a huge number of terms, which is convenient to gather according to their eigenvalues of the counting operator $\mathcal{N}_\varepsilon \equiv \bar{\varepsilon}^i \dfrac{\partial}{\partial \bar{\varepsilon}^i}$

$$\Delta = \sum_{n=0}^{3} \Delta^{(n)} \ , \tag{3.23}$$

with

$$[\mathcal{N}_\varepsilon, \Delta^{(n)}] = n\Delta^{(n)} \ . \tag{3.24}$$

Notice that for power counting reasons it is not possible to write a local integrated functional with the right quantum numbers and having $n \geq 4$

$$\Delta^{(n)} = 0 \qquad \text{for } n \geq 4 \ . \tag{3.25}$$

We must act on the functional Δ with the operator $\mathcal{B}_\Sigma$, which accordingly writes

$$\mathcal{B}_\Sigma = \sum_{n=0}^{2} s^{(n)} \ , \tag{3.26}$$

with

$$[\mathcal{N}_\varepsilon, s^{(n)}] = n s^{(n)} \ . \tag{3.27}$$

The explicit form of $\Delta^{(n)}$ and $s^{(n)}$ is given in the appendix.

The Wess–Zumino consistency condition (3.20), splitted into eigenstates of the operator $\mathcal{N}_\varepsilon$, reads

$$\begin{aligned}
s^{(0)}\Delta^{(0)} &= 0 \\
s^{(0)}\Delta^{(1)} + s^{(1)}\Delta^{(0)} &= 0 \\
s^{(0)}\Delta^{(2)} + s^{(1)}\Delta^{(1)} + s^{(2)}\Delta^{(0)} &= 0 \\
s^{(0)}\Delta^{(3)} + s^{(1)}\Delta^{(2)} + s^{(2)}\Delta^{(1)} &= 0 \\
s^{(1)}\Delta^{(3)} + s^{(2)}\Delta^{(2)} &= 0 \\
s^{(2)}\Delta^{(3)} &= 0
\end{aligned} \tag{3.28}$$

It is both an easy and long algebraic exercise to solve the equations (3.28) and to verify that finally the solutions $\Delta^{(n)}$ are such that the whole breaking Δ is a $\mathcal{B}_\Sigma$–variation

$$\Delta = \mathcal{B}_\Sigma \widehat{\Delta} \ . \tag{3.29}$$

Notice that the result (3.29) states that for $N=2$ SYMs there are no anomalies already at algebraic level, contrarily to what happens for ordinary YM theories and for $N=1$

SYMs, where the Adler–Bardeen anomaly [26], or its supersymmetric extension [10, 27], obeys the consistency condition (3.20), and whose absence is guaranteed by the vanishing of its coefficient [28].

Therefore the Slavnov identity (2.7) can be implemented to all orders of perturbation theory. This, together with the form (3.12) of the counterterm, implies the renormalizability of $N = 2$ SYMs.

4 Conclusions

We proved the renormalizability of $N = 2$ SYMs in a purely algebraic way, *i.e.* without assuming the existence of any regularization scheme. The most general counterterm (3.12) compatible with all the symmetries of the theory can be reabsorbed by renormalizations of the only coupling constant g^2 and of the quantum fields belonging to the vector and matter multiplet, the ghost field not renormalizing as a consequence of the Landau gauge choice. This algebraic result reflects the fact that in general $N = 2$ SYMs do get divergent quantum corrections. It is known, on the other hand [4, 8], that only those theories verifying the condition (4.1) are finite :

$$\sum_\sigma m_\sigma T(R_\sigma) = C_2(G) \ , \tag{4.1}$$

where m_σ is the number of $N=2$ matter multiplets in the representation R_σ and the Casimir and Dynkin indices $C_2,\ T$ are defined as usual by

$$\begin{aligned} C_2(G)\delta^{ab} = &\ f^{amn}f^{bmn} \\ T(R_\sigma)\delta^{ab} = &\ \mathrm{Tr}(T^aT^b)\ . \end{aligned} \tag{4.2}$$

The selection rule (4.1) has been obtained following two arguments. The first of them [4] makes use of a particular superfield formulation of $N=2$ theories regulated by introducing higher derivatives, which do not regulate the one loop contribution to the quantum vertex functional Γ. The second argument [8] works in the superspace background field formalism of N–extended supersymmetric theories, where no contribution to Γ above one loop is possible. The nonrenormalization condition (4.1) corresponds to the vanishing at one loop, and hence at all orders, of the β–function of the coupling constant, and it has not been reproduced yet in the general framework of an algebraic analysis, relying only on the principles of locality and power counting. For this purpose, our work may be the starting point to extend to $N=2$ SYMs the one–loop *criteria* given in [21] for the finiteness of $N=1$ supersymmetric gauge theories. Notice that the $N=4$ theory, interpreted as a $N=2$ with matter in the adjoint representation, trivially satisfies the condition (4.1), since $m_\sigma=1\ ,\ T(R_\sigma)=C_2(G)$.

The second part of the renormalization of $N=2$ SYMs consisted in the verification that none of the symmetries forming the supersymmetry algebra are anomalous and therefore hold good also at the quantum level. This result has been achieved by exploiting the formulation given in [15], according to which the whole analysis is reduced to the solution

of the cohomology problem (3.20) of the generalized Slavnov operator (2.8). In particular no $N = 2$ supersymmetric extension of the Adler–Bardeen gauge anomaly does exist.

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A Appendix

A.1 Linearized Slavnov operator

In the space of functionals depending on the fields listed in Table 1, the linearized Slavnov operator is modified as follows [15]

$$\hat{\mathcal{B}}_\Sigma = \int d^4x \bigg(\frac{\delta\hat\Sigma}{\delta\eta^{a\mu}}\frac{\delta}{\delta A^a_\mu} + \frac{\delta\hat\Sigma}{\delta A^a_\mu}\frac{\delta}{\delta\Omega^{a\mu}} + \frac{\delta\hat\Sigma}{\delta L^a}\frac{\delta}{\delta c^a} + \frac{\delta\hat\Sigma}{\delta c^a}\frac{\delta}{\delta L^a} + \frac{\delta\hat\Sigma}{\delta M^a}\frac{\delta}{\delta A^a} + \frac{\delta\hat\Sigma}{\delta A^a}\frac{\delta}{\delta M^a} \tag{A.1}$$
$$+\frac{\delta\hat\Sigma}{\delta N^a}\frac{\delta}{\delta B^a} + \frac{\delta\hat\Sigma}{\delta B^a}\frac{\delta}{\delta N^a} + \frac{\delta\hat\Sigma}{\delta\bar\Lambda^{ai}}\frac{\delta}{\delta\lambda^a_i} - \frac{\delta\hat\Sigma}{\delta\lambda^a_i}\frac{\delta}{\delta\bar\Lambda^{ai}} + \frac{\delta\hat\Sigma}{\delta U^*_{iA}}\frac{\delta}{\delta A^{iA}} + \frac{\delta\hat\Sigma}{\delta A^{iA}}\frac{\delta}{\delta U^*_{iA}}$$
$$+\frac{\delta\hat\Sigma}{\delta U^{iA}}\frac{\delta}{\delta A^*_{iA}} + \frac{\delta\hat\Sigma}{\delta A^*_{iA}}\frac{\delta}{\delta U^{iA}} + \frac{\delta\hat\Sigma}{\delta\bar\Psi_A}\frac{\delta}{\delta\psi^A} - \frac{\delta\hat\Sigma}{\delta\psi^A}\frac{\delta}{\delta\bar\Psi_A} + \frac{\delta\hat\Sigma}{\delta\Psi^A}\frac{\delta}{\delta\bar\psi_A} - \frac{\delta\hat\Sigma}{\delta\bar\psi_A}\frac{\delta}{\delta\Psi^A}\bigg)$$

By filtrating with the counting operator $\mathcal{N}_\varepsilon$, $\hat{\mathcal{B}}_\Sigma$ decomposes as (3.26). Explicitely we have

$$\begin{aligned}
s^{(0)}A^a &= f^{abc}c^bA^c \\
s^{(0)}B^a &= f^{abc}c^bB^c \\
s^{(0)}A^a_\mu &= -(D_\mu c)^a \\
s^{(0)}\lambda^a_{\alpha i} &= f^{abc}c^b\lambda^c_{\alpha i} \\
s^{(0)}c^a &= \frac{1}{2}f^{abc}c^bc^c \\
s^{(0)}A^{iA} &= (T^a)^A_B c^a A^{iB} \\
s^{(0)}\psi^A_\alpha &= (T^a)^A_B c^a \psi^B_\alpha
\end{aligned}$$

$$
\begin{aligned}
s^{(0)}L^a &= f^{abc}c^bL^c + f^{abc}M^bA^c + f^{abc}N^bB^c - (D\eta)^a + f^{abc}(\bar{\Lambda}^{bi}\lambda_i^c) - (T^a)^A_B U^*_{iA}A^{iB} \\
&\quad -(T^a)^A_B(\bar{\Psi}_A\psi^B) + (T^a)^A_B U^{iB}A^*_{iA} - (T^a)^A_B(\bar{\psi}_A\Psi^B) \qquad \text{(A.2)} \\
s^{(0)}M^a &= -\frac{1}{g^2}(D^2A)^a + \frac{1}{2g^2}f^{abc}(\bar{\lambda}^{bi}\lambda_i^c) - \frac{1}{g^2}f^{abc}B^bf^{cmn}A^mB^n \\
&\quad +[(T^a)^A_D(T^b)^D_B + (T^b)^A_D(T^a)^D_B]A^bA^*_{iA}A^{iB} - \frac{1}{2}(T^a)^A_B(\bar{\psi}_A\psi^B) + f^{abc}c^bM^c \\
s^{(0)}N^a &= -\frac{1}{g^2}(D^2B)^a - \frac{i}{2g^2}f^{abc}(\bar{\lambda}^{bi}\gamma_5\lambda_i^c) + \frac{1}{g^2}f^{abc}A^bf^{cmn}A^mB^n \\
&\quad +[(T^a)^A_D(T^b)^D_B + (T^b)^A_D(T^a)^D_B]B^bA^*_{iA}A^{iB} - i\frac{1}{2}(T^a)^A_B(\bar{\psi}_A\gamma_5\psi^B) + f^{abc}c^bN^c \\
s^{(0)}\eta^{a\mu} &= -\frac{1}{g^2}(D_\nu F^{\mu\nu})^a + \frac{1}{g^2}f^{abc}A^b(D^\mu A)^c + \frac{1}{g^2}f^{abc}B^b(D^\mu B)^c - \frac{1}{2g^2}f^{abc}(\bar{\lambda}^{bi}\gamma^\mu\lambda_i^c) \\
&\quad +(T^a)^A_B A^{iB}(D^\mu A^*_i)_A - (T^a)^A_B A^*_{iA}(D^\mu A^i)^B - \frac{1}{2}(T^a)^A_B(\bar{\psi}_A\gamma^\mu\psi^B) + f^{abc}c^b\eta^{c\mu} \\
s^{(0)}\bar{\Lambda}^{a\alpha i} &= -(D_\mu\bar{\lambda}^i\gamma^\mu)^{a\alpha i} - f^{abc}\bar{\lambda}^{b\alpha i}A^c + if^{abc}(\bar{\lambda}^{bi}\gamma_5)^\alpha B^c - (T^a)^A_B\bar{\psi}^\alpha_A A^{iB} \\
&\quad +(T^a)^A_B(i\gamma_5C\psi^B)^\alpha\epsilon^{ij}A^*_{jA} + f^{abc}\bar{\Lambda}^{b\alpha i}c^c \\
s^{(0)}U^{iA} &= -(D^2A^i)^A + (T^a)^A_B(\bar{\lambda}^{ai}\psi^B) + (T^a)^A_D(T^b)^D_B(A^aA^b + B^aB^b)A^{iB} - (T^a)^A_B U^{iB}c^a \\
s^{(0)}\Psi^A_\alpha &= \frac{1}{2}(\gamma^\mu D_\mu\psi)^A_\alpha + (T^a)^A_B\lambda^a_{\alpha i} + i\frac{1}{2}(T^a)^A_B(\gamma_5\psi^B)B^a \\
&\quad +\frac{1}{2}(T^a)^A_B\psi^B_\alpha A^a + (T^a)^A_B c^a\psi^B_\alpha
\end{aligned}
$$

$$
\begin{aligned}
s^{(1)}A^a &= \bar{\varepsilon}^i\lambda_i^a \\
s^{(1)}B^a &= i\bar{\varepsilon}^i\gamma_5\lambda_i^a \\
s^{(1)}A^a_\mu &= \bar{\varepsilon}^i\gamma_\mu\lambda_i^a \\
s^{(1)}\lambda^a_{\alpha i} &= \frac{1}{2}F^a_{\mu\nu}(\sigma^{\mu\nu}\varepsilon_i)_\alpha - (D_\mu A)^a(\gamma^\mu\varepsilon_i)_\alpha + i(D_\mu B)^a(\gamma^\mu\gamma_5\varepsilon_i)_\alpha + if^{abc}A^bB^c(\gamma_5\varepsilon_i)_\alpha \\
s^{(1)}c^a &= 0 \\
s^{(1)}A^{iA} &= \bar{\varepsilon}^i\psi^A \\
s^{(1)}\psi^A_\alpha &= -2(D_\mu A^i)^A(\gamma^\mu\varepsilon_i)_\alpha + 2(T^a)^A_B A^{iB}A^a\varepsilon_{\alpha i} - 2i(T^a)^A_B A^{iB}B^a(\gamma_5\varepsilon_i)_\alpha \\
s^{(1)}L^a &= 0 \qquad \text{(A.3)} \\
s^{(1)}M^a &= (D_\mu\bar{\Lambda}^i)^a\gamma^\mu\varepsilon_i - if^{abc}(\bar{\Lambda}^{bi}\gamma_5\varepsilon_i)B^c + 2(T^a)^A_B(\bar{\Psi}_A\varepsilon_i)A^{iB} + 2(T^a)^A_B A^*_{iA}(\bar{\varepsilon}^i\Psi^B) \\
s^{(1)}N^a &= -i(D_\mu\bar{\Lambda}^i)^a\gamma^\mu\gamma_5\varepsilon_i + if^{abc}(\bar{\Lambda}^{bi}\gamma_5\varepsilon_i)A^c - 2i(T^a)^A_B(\bar{\Psi}_A\gamma_5\varepsilon_i)A^{iB} \\
&\quad -2i(T^a)^A_B A^*_{iA}(\bar{\varepsilon}^i\gamma_5\Psi^B) \\
s^{(1)}\eta^{a\mu} &= (D_\nu\bar{\Lambda}^i)^a\sigma^{\mu\nu}\varepsilon_i + f^{abc}(\bar{\Lambda}^{bi}\gamma^\mu\varepsilon_i)A^c - if^{abc}(\bar{\Lambda}^{bi}\gamma^\mu\gamma_5\varepsilon_i)B^c \\
&\quad -2(T^a)^A_B(\bar{\Psi}_A\gamma^\mu\varepsilon_i)A^{iB} - 2(T^a)^A_B A^*_{iB}(\bar{\varepsilon}^i\gamma^\mu\Psi^A) \\
s^{(1)}\bar{\Lambda}^{a\alpha i} &= M^a\bar{\varepsilon}^i + iN^a(\bar{\varepsilon}^i\gamma^5)^\alpha + \eta^{a\mu}(\bar{\varepsilon}^i\gamma^\mu)^\alpha \\
s^{(1)}U^{iA} &= -2(\bar{\varepsilon}^i\gamma^\mu D_\mu\Psi)^A + 2(T^a)^A_B A^a(\bar{\varepsilon}^i\Psi^B) - 2i(T^a)^A_B B^a(\bar{\varepsilon}^i\gamma_5\Psi^B) \\
s^{(1)}\Psi^A_\alpha &= -U^{iA}\varepsilon_{\alpha i}
\end{aligned}
$$

$$
\begin{aligned}
s^{(2)}A^a &= 0 \\
s^{(2)}B^a &= 0 \\
s^{(2)}A^a_\mu &= 0 \\
s^{(2)}\lambda^a_{\alpha i} &= -2(\bar\varepsilon^j\Lambda^a_i)\varepsilon_{\alpha j} + (\bar\varepsilon^j\Lambda^a_j)\varepsilon_{\alpha i} \\
s^{(2)}c^a &= -(\bar\varepsilon^i\gamma^\mu\varepsilon_i)A^a_\mu - i(\bar\varepsilon^i\gamma_5\varepsilon_i)B^a + (\bar\varepsilon^i\varepsilon_i)A^a \\
s^{(2)}A^{iA} &= 0 \\
s^{(2)}\psi^A_\alpha &= -(\bar\varepsilon^i\varepsilon_i)\Psi^A_\alpha + (\bar\varepsilon^i\gamma_5\varepsilon_i)(\gamma_5\Psi^A)_\alpha + (\bar\varepsilon^i\gamma^\mu\varepsilon_i)(\gamma_\mu\Psi^A)_\alpha \\
s^{(2)}L^a &= 0 \\
s^{(2)}M^a &= (\bar\varepsilon^i\varepsilon_i)L^a \\
s^{(2)}N^a &= -i(\bar\varepsilon^i\gamma_5\varepsilon_i)L^a \\
s^{(2)}\eta^{a\mu} &= -(\bar\varepsilon^i\gamma^\mu\varepsilon_i)L^a \\
s^{(2)}\bar\Lambda^{a\alpha i} &= 0 \\
s^{(2)}\Psi^A_\alpha &= 0
\end{aligned}
\tag{A.4}
$$

A.2 Counterterm

The most general candidate for the counterterm is $\Sigma_c = \Sigma_c^{(0)} + \Sigma_c^{(1)} + \Sigma_c^{(2)}$, where

$$
\begin{aligned}
\Sigma_c^{(0)} = \int d^4x \Big(& a_1 F^{\mu\nu}F^{a\mu\nu} + a_2 f^{abc}(\bar\lambda^{bi}\lambda^c_i)A^a + a_3 f^{abc}(\bar\lambda^{bi}\gamma_5\lambda^c_i)B^a + a_4\bar\lambda^{ai}\gamma^\mu(D_\mu\lambda_i)^a \\
&+ a_5(T^a)^A_B(\bar\lambda^{ai}\psi^B)A^*_{iA} + a_6(T^a)^A_B(\bar\psi_A\lambda^a_i)A^{iB} + a_7(T^a)^A_B A^a(\bar\psi_A\psi^B) \\
&+ a_8(T^a)^A_B B^a(\bar\psi_A\gamma_5\psi^b) + a_9\bar\psi_A\gamma^\mu(D_\mu\psi)^A + a_{10}(D^\mu A)^a(D_\mu A)^a \\
&+ a_{11}(D^\mu B)^a(D_\mu B)^a + a_{12}f^{amn}A^mB^nf^{apq}A^pB^q + a_{13}(D^\mu A^i)^A(D_\mu A^*_i)_A \\
&+ a_{14}(T^a)^A_D(T^b)^D_B A^aA^bA^*_{iA}A^{iB} + a_{15}(T^a)^A_D(T^b)^D_B B^aB^bA^*_{iA}A^{iB} + a_{16}c^a_\mu\eta^{a\mu} \\
&+ a_{17}D^{abcd}A^aA^bA^cA^d + a_{18}D^{abcd}B^aB^bB^cB^d \Big)
\end{aligned}
\tag{A.5}
$$

$$
\begin{aligned}
\Sigma_c^{(1)} = \int d^4x \Big(& b_1(\bar\varepsilon^i\lambda^a_i)M^a + b_2(\bar\varepsilon^i\gamma_5\lambda^a_i)N^a + b_3(\bar\varepsilon^i\gamma_\mu\lambda^a_i)\eta^{a\mu} + b_4(\bar\Lambda^{ai}\gamma^\mu\varepsilon_i)(D_\mu A)^a \\
&+ b_5(\bar\Lambda^{ai}\gamma^\mu\gamma_5\varepsilon_i)(D_\mu B)^a + b_6(\bar\Lambda^{ai}\sigma^{\mu\nu}\varepsilon_i)F^a_{\mu\nu} + b_7(\bar\Lambda^{ai}\varepsilon_i)\partial A^a \\
&+ b_8 f^{abc}(\bar\Lambda^{ai}\gamma_5\varepsilon_i)A^bB^c + b_9(T^a)^A_B(\bar\Lambda^{ai}\varepsilon_i)A^{jB}A^*_{jA} + b_{10}(T^a)^A_B(\bar\Lambda^{aj}\varepsilon_i)A^{jB}A^*_{iA} \Big)
\end{aligned}
\tag{A.6}
$$

$$
\Sigma_c^{(2)} = \int d^4x \Big(c_1(\bar\varepsilon^i\varepsilon_i)L^aA^a + c_2(\bar\varepsilon^i\gamma_5\varepsilon_i)L^aB^a + c_3(\bar\varepsilon^i\gamma^\mu\varepsilon_i)L^aA^a_\mu
\tag{A.7}
$$

$$+c_{4_R}(\bar{\varepsilon}^i\gamma_R\Lambda_i^a)(\bar{\Lambda}^{aj}\gamma_R\varepsilon_j)+c_{5_R}(\bar{\varepsilon}^i\gamma_R\Lambda_j^a)(\bar{\Lambda}^{aj}\gamma_R\varepsilon_i)+c_{6_R}(\bar{\varepsilon}^i\gamma_R\Psi^A)(\bar{\Psi}_A\gamma_R\varepsilon_i)\Big)\ ,$$

where $\gamma_R \in \{\mathbb{1}, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}\}$ and D^{abcd} is the completely symmetric invariant tensor of rank four

$$D^{abcd} \equiv d^{nab}f^{ncd} + d^{nac}f^{ndb} + d^{nad}f^{nbc} \tag{A.8}$$

A.3 Anomaly

The most general candidate for the anomaly is $\Delta = \Delta^{(0)} + \Delta^{(1)} + \Delta^{(2)} + \Delta^{(3)}$, where

$$\Delta^{(0)} = \alpha_0 \int d^4x\, \epsilon^{\mu\nu\rho\sigma} c_\mu^a \Big(d^{abc}(\partial_\nu A_\rho^b)A_\sigma^c + \frac{1}{12}D^{abcd}A_\nu^b A_\rho^c A_\sigma^d \Big) + s^{(0)}\Sigma_c^{(0)} \tag{A.9}$$

$$\begin{aligned}
&\Delta^{(1)} = \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (\mathrm{A.10})\\
&\int d^4x \Big(\alpha_1(\bar{\Lambda}^{ai}\varepsilon_i)\partial^2 c^a + \alpha_2^{abc}(\bar{\Lambda}^{ai}\gamma^\mu\varepsilon_i)c_\mu^b A^c + \alpha_3^{abc}(\bar{\Lambda}^{ai}\gamma^\mu\gamma_5\varepsilon_i)c_\mu^b B^c \\
&+\alpha_4^{abc}(\bar{\Lambda}^{ai}\varepsilon_i)A_\mu^b c^{c\mu} + \alpha_5^{abc}(\bar{\Lambda}^{ai}\sigma^{\mu\nu}\varepsilon_i)A_\mu^b c_\nu^c + \alpha_6(T^a)_B^A(\bar{\varepsilon}^i\gamma^\mu\Psi^A)c_\mu^a A_{iA}^* \\
&+\alpha_7(T^a)_B^A(\bar{\Psi}_A\gamma^\mu\varepsilon_i)c_\mu^a A^{iB} + \alpha_8(\bar{\varepsilon}^i\lambda_i^a)\partial^2 A^a + \alpha_9(\bar{\varepsilon}^i\gamma_5\lambda_i^a)\partial^2 B^a \\
&+\alpha_{10}(\bar{\varepsilon}^i\gamma^\mu\lambda_i^a)\partial_\mu\partial A^a + \alpha_{11}(\bar{\varepsilon}^i\gamma^\mu\lambda_i^a)\partial^2 A_\mu^a + \alpha_{12}(\bar{\varepsilon}^i\psi^A)\partial^2 A_{iA}^* \\
&+\alpha_{13}(\bar{\psi}_A\varepsilon_i)\partial^2 A^{iA} + \alpha_{14}^{abc}(\bar{\varepsilon}^i\gamma^\mu\lambda_i^a)(\partial_\mu A^b)A^c + \alpha_{15}^{abc}(\bar{\varepsilon}^i\gamma^\mu\gamma_5\lambda_i^a)(\partial_\mu A^b)B^c \\
&+\alpha_{16}^{abc}(\bar{\varepsilon}^i\gamma^\mu\gamma_5\lambda_i^a)A^b(\partial_\mu B^c) + \alpha_{17}^{abc}(\bar{\varepsilon}^i\lambda_i^a)(\partial A^b)A^c + \alpha_{18}^{abc}(\bar{\varepsilon}^i\lambda_i^a)A_\mu^b(\partial^\mu A^c) \\
&+\alpha_{19}^{abc}(\bar{\varepsilon}^i\sigma^{\mu\nu}\lambda_i^a)(\partial_\mu A_\nu^b)A^c + \alpha_{20}^{abc}(\bar{\varepsilon}^i\sigma^{\mu\nu}\lambda_i^a)A_\nu^b(\partial_\mu A^c) + \alpha_{21}(T^a)_B^A(\bar{\varepsilon}^i\gamma^\mu\psi^A)(\partial_\mu A^a)A_{iA}^* \\
&+\alpha_{22}(T^a)_B^A(\bar{\varepsilon}^i\gamma^\mu\psi^A)A^a(\partial_\mu A_{iA}^*) + \alpha_{23}(T^a)_B^A(\bar{\psi}_A\gamma^\mu\varepsilon_i)(\partial_\mu A^a)A^{iB} \\
&+\alpha_{24}(T^a)_B^A(\bar{\psi}_A\gamma^\mu\varepsilon_i)A^a(\partial_\mu A^{iB}) + \alpha_{25}^{abc}(\bar{\varepsilon}^i\gamma^\mu\lambda_i^a)(\partial_\mu B^b)B^c \\
&+\alpha_{26}^{abc}(\bar{\varepsilon}^i\gamma_5\lambda_i^a)(\partial A^b)B^c + \alpha_{27}^{abc}(\bar{\varepsilon}^i\gamma_5\lambda_i^a)A_\mu^b(\partial^\mu B^c) \\
&+\alpha_{28}^{abc}(\bar{\varepsilon}^i\sigma^{\mu\nu}\gamma_5\lambda_i^a)(\partial_\mu A_\nu^b)B^c + \alpha_{29}^{abc}(\bar{\varepsilon}^i\sigma^{\mu\nu}\gamma_5\lambda_i^a)A_\mu^b(\partial_\nu B^c) \\
&+\alpha_{30}(T^a)_B^A(\bar{\varepsilon}^i\gamma^\mu\gamma_5\psi^B)(\partial_\mu B^a)A_{iA}^* + \alpha_{31}(T^a)_B^A(\bar{\varepsilon}^i\gamma^\mu\gamma_5\psi^B)B^a(\partial_\mu A_{iA}^*) \\
&+\alpha_{32}(T^a)_B^A(\bar{\psi}_A\gamma^\mu\gamma_5\varepsilon_i)(\partial_\mu B^a)A^{iB} + \alpha_{33}(T^a)_B^A(\bar{\psi}_A\gamma^\mu\gamma_5\varepsilon_i)B^a(\partial_\mu A^{iB}) \\
&+\alpha_{34}^{abc}(\bar{\varepsilon}^i\gamma^\mu\lambda_i^a)(\partial_\mu A_\nu^b)A_\nu^c + \alpha_{35}^{abc}(\bar{\varepsilon}^a\gamma^\mu\lambda_i^a)(\partial^\nu A_\mu^b)A_\nu^c \\
&+\alpha_{36}^{abc}(\bar{\varepsilon}^i\gamma^\mu\lambda_i^a)A_\mu^b(\partial A^c) + \alpha_{37}^{abc}\epsilon^{\mu\nu\rho\sigma}(\bar{\varepsilon}^i\gamma_\mu\lambda_i^a)(\partial_\nu A_\rho^b)A_\sigma^c \\
&+\alpha_{38}(T^a)_B^A(\bar{\varepsilon}^i\psi^B)(\partial A^a)A_{iA}^* + \alpha_{39}(T^a)_B^A(\bar{\varepsilon}^i\psi^B)A_\mu^a(\partial^\mu A_{iA}^*) \\
&+\alpha_{40}(T^a)_B^A(\bar{\psi}_A\varepsilon_i)(\partial A^a)A^{iB} + \alpha_{41}(T^a)_B^A(\bar{\psi}_A\varepsilon_i)A_\mu^a(\partial^\mu A^{iB}) \\
&+\alpha_{42}(T^a)_B^A(\bar{\varepsilon}^i\sigma^{\mu\nu}\psi^B)(\partial_\mu A_\nu^a)A_{iA}^* + \alpha_{43}(T^a)_B^A(\bar{\varepsilon}^i\sigma^{\mu\nu}\psi^B)A_\mu^a(\partial_\nu A_{iA}^*) \\
&+\alpha_{44}(T^a)_B^A(\bar{\psi}_A\sigma^{\mu\nu}\varepsilon_i)(\partial_\mu A_\nu^a)A^{iB} + \alpha_{45}(T^a)_B^A(\bar{\psi}_A\sigma^{\mu\nu}\varepsilon_i)A_\mu^a(\partial_\nu A^{iB}) \\
&+\alpha_{46}(T^a)_B^A(\bar{\varepsilon}^i\gamma^\mu\lambda_i^a)(\partial_\mu A^{iB})A_{iA}^* + \alpha_{47}(T^a)_B^A(\bar{\varepsilon}^i\gamma^\mu\lambda_i^a)A^{iB}(\partial_\mu A_{iB}^*)
\end{aligned}$$

$$
\begin{aligned}
&+\alpha_{48}^{abcd}(\bar{\varepsilon}^i\lambda_i^a)A^bA^cA^d+\alpha_{49}^{abcd}(\bar{\varepsilon}^i\gamma_5\lambda_i^a)A^bA^cB^d\\
&+\alpha_{50}^{abcd}(\bar{\varepsilon}^i\gamma^\mu\lambda_i^a)A_\mu^bA^cA^d+\alpha_{51}(T^a)_D^A(T^b)_B^DA^aA^b(\bar{\varepsilon}^i\psi^B)A_{iA}^*\\
&+\alpha_{52}(T^a)_D^A(T^b)_B^D(\bar{\psi}_A\varepsilon_i)A^{iB}A^aA^b+\alpha_{53}^{abcd}(\bar{\varepsilon}^i\lambda_i^a)A^bB^cB^d\\
&+\alpha_{54}^{abcd}(\bar{\varepsilon}^i\gamma^\mu\gamma_5\lambda_i^a)A_\mu^bA^cB^d+\alpha_{55}(T^a)_D^A(T^b)_B^DA^aB^b(\bar{\varepsilon}^i\gamma_5\psi^B)A_{iA}^*\\
&+\alpha_{56}(T^a)_D^A(T^b)_B^D(\bar{\psi}_A\gamma_5\varepsilon_i)A^{iB}A^aB^b+\alpha_{57}^{abcd}(\bar{\varepsilon}^i\lambda_i^a)A^bA^{c\mu}A_\mu^d\\
&+\alpha_{58}^{abcd}(\bar{\varepsilon}^i\sigma^{\mu\nu}\lambda_i^a)A^bA_\mu^cA_\nu^d+\alpha_{59}(T^a)_D^A(T^b)_B^D(\bar{\varepsilon}^i\gamma^\mu\psi^B)A_{iA}^*A_\mu^aA^b\\
&+\alpha_{60}(T^a)_D^A(T^b)_B^D(\bar{\psi}_A\gamma^\mu\varepsilon_i)A^{iB}A_\mu^aA^b+\alpha_{61}(T^a)_D^A(T^b)_B^D(\bar{\varepsilon}^i\lambda_i^a)A^bA^{iB}A_{iA}^*\\
&+\alpha_{62}^{abcd}(\bar{\varepsilon}^i\gamma_5\lambda_i^a)B^bB^cB^d+\alpha_{63}^{abcd}(\bar{\varepsilon}^i\gamma^\mu\lambda_i^a)A_\mu^bB^cB^d\\
&+\alpha_{64}(T^a)_D^A(T^b)_B^D(\bar{\varepsilon}^i\psi^B)A_{iA}^*B^aB^b+\alpha_{65}(T^a)_D^A(T^b)_B^D(\bar{\psi}_A\varepsilon_i)A^{iB}B^aB^b\\
&+\alpha_{66}^{abcd}(\bar{\varepsilon}^i\gamma_5\lambda_i^a)A_\mu^bA^{c\mu}B^d+\alpha_{67}^{abcd}(\bar{\varepsilon}^i\sigma^{\mu\nu}\gamma_5\lambda_i^a)A_\mu^bA_\nu^cB^d\\
&+\alpha_{68}(T^a)_D^A(T^b)_B^D(\bar{\varepsilon}^i\gamma^\mu\gamma_5\psi^B)A_{iA}^*A_\mu^aB^b+\alpha_{69}(T^a)_D^A(T^b)_B^D(\bar{\psi}_A\gamma^\mu\gamma_5\varepsilon_i)A^{iB}A_\mu^aB^b\\
&+\alpha_{70}(T^a)_D^A(T^b)_B^D(\bar{\varepsilon}^i\gamma_5\lambda_i^a)B^bA_{iA}^*A^{iB}+\alpha_{71}^{abcd}(\bar{\varepsilon}^i\gamma^\mu\lambda_i^a)A_\mu^bA^{c\nu}A_\nu^d\\
&+\alpha_{72}^{abcd}\epsilon^{\mu\nu\rho\sigma}(\bar{\varepsilon}^i\gamma_\mu\lambda_i^a)A_\nu^bA_\rho^cA_\sigma^d+\alpha_{73}(T^a)_D^A(T^b)_B^D(\bar{\varepsilon}^i\psi^B)A_{iA}^*A^{a\mu}A_\mu^b\\
&+\alpha_{74}(T^a)_D^A(T^b)_B^D(\bar{\psi}_A\varepsilon_i)A^{iB}A^{a\mu}A_\mu^b+\alpha_{75}(T^a)_D^A(T^b)_B^D(\bar{\varepsilon}^i\sigma^{\mu\nu}\psi^B)A_{iA}^*A_\mu^aA_\nu^b\\
&+\alpha_{76}(T^a)_D^A(T^b)_B^D(\bar{\psi}_A\sigma^{\mu\nu}\varepsilon_i)A^{iB}A_\mu^aA_\nu^b+\alpha_{77}(T^a)_D^A(T^b)_B^D(\bar{\varepsilon}^i\gamma^\mu\lambda_i^a)A_\mu^bA_{iA}^*A^{iB}\\
&+\alpha_{78}A^aA^a(\bar{\varepsilon}^i\psi^A)A_{iA}^*+\alpha_{79}A^aA^a(\bar{\psi}_A\varepsilon_i)A^{iA}\\
&+\alpha_{80}A^aB^a(\bar{\varepsilon}^i\gamma_5\psi^A)A_{iA}^*+\alpha_{81}(\bar{\psi}_A\gamma_5\varepsilon_i)A^{iA}A^aB^a\\
&+\alpha_{82}(\bar{\varepsilon}^i\gamma^\mu\psi^A)A_{iA}^*A_\mu^aA^a+\alpha_{83}(\bar{\psi}_a\gamma^\mu\varepsilon_i)A^{iA}A_\mu^aA^a+\alpha_{84}(\bar{\varepsilon}^i\lambda_i^a)A^aA^{jA}A_{jA}^*\\
&+\alpha_{85}(\bar{\varepsilon}^i\psi^A)A_{iA}^*B^aB^a+\alpha_{86}(\bar{\psi}_A\varepsilon_i)A^{iA}B^aB^a+\alpha_{87}(\bar{\varepsilon}^i\gamma^\mu\gamma_5\psi^A)A_{iA}^*A_\mu^aB^a\\
&+\alpha_{88}(\bar{\psi}_A\gamma^\mu\gamma_5\varepsilon_i)A^{iA}A_\mu^aB^a+\alpha_{89}(\bar{\varepsilon}^i\gamma_5\lambda_i^a)B^aA_{jA}^*A^{jA}+\alpha_{90}(\bar{\varepsilon}^i\psi^A)A_{iA}^*A^{a\mu}A_\mu^a\\
&+\alpha_{91}(\bar{\psi}_A\varepsilon_i)A^{iA}A^{a\mu}A_\mu^a+\alpha_{92}(\bar{\varepsilon}^i\gamma^\mu\lambda_i^a)A_\mu^aA_{jA}^*A^{jA}+\alpha_{93}(T^a)_D^A(T^b)_B^D(\bar{\varepsilon}^i\lambda_j^a)A^bA_{iA}^*A^{jB}\\
&+\alpha_{94}(\bar{\varepsilon}^i\lambda_j^a)A^aA_{iA}^*A^{jA}+\alpha_{95}(T^a)_D^A(T^b)_B^D(\bar{\lambda}^{ai}\varepsilon_j)A^bA_{iA}^*A^{jB}+\alpha_{96}(\bar{\lambda}^{ai}\varepsilon_j)A^aA_{iA}^*A^{jA}\\
&+\alpha_{97}(T^a)_D^A(T^b)_B^D(\bar{\varepsilon}^i\gamma_5\lambda_j^a)B^bA_{iA}^*A^{jB}+\alpha_{98}(\bar{\varepsilon}^i\gamma_5\lambda_j^a)B^aA_{iA}^*A^{jB}\\
&+\alpha_{99}(T^a)_D^A(T^b)_B^D(\bar{\lambda}^{ai}\gamma_5\varepsilon_j)B^bA_{iA}^*A^{jB}+\alpha_{100}(\bar{\lambda}^{ai}\gamma_5\varepsilon_j)B^aA_{iA}^*A^{jA}\\
&+\alpha_{101}(T^a)_D^A(T^b)_B^D(\bar{\varepsilon}^i\gamma^\mu\lambda_j^a)A_\mu^bA_{iA}^*A^{jB}+\alpha_{102}(\bar{\varepsilon}^i\gamma^\mu\lambda_j^a)A_\mu^aA_{iA}^*A^{jA}\\
&+\alpha_{103}(T^a)_D^A(T^b)_B^D(\bar{\lambda}^{aj}\gamma^\mu\varepsilon_i)A_\mu^bA_{jA}^*A^{iB}+\alpha_{104}(\bar{\lambda}^{aj}\gamma^\mu\varepsilon_i)A_\mu^bA_{jA}^*A^{iA}\\
&+\alpha_{105}T_{AD}^{BC}(\bar{\varepsilon}^i\psi^A)A_{iB}^*A_{jC}^*A^{jD}+\alpha_{106}T_{AD}^{BC}(\bar{\psi}_B\varepsilon_i)A^{iA}A_{jC}^*A^{jD}\\
&+\alpha_{107}(T^a)_B^A(\bar{\lambda}^{ai}\gamma^\mu\varepsilon_j)(\partial_\mu A^{jB})A_{iA}^*+\alpha_{108}(T^a)_B^A(\bar{\lambda}^{ai}\gamma^\mu\varepsilon_j)A^{jB}(\partial_\mu A_{iA}^*)\\
&+\alpha_{109}(T^a)_B^A(\bar{\varepsilon}^i\gamma^\mu\lambda_j^a)(\partial_\mu A^{jB})A_{iA}^*+\alpha_{110}(T^a)_B^A(\bar{\varepsilon}^i\gamma^\mu\lambda_j^a)A^{jB}(\partial_\mu A_{iA}^*)\\
&+\alpha_{111}f^{abc}(\bar{\varepsilon}^i\lambda_i^a)(\bar{\lambda}^{bj}\lambda_j^c)+\alpha_{112}f^{abc}(\bar{\varepsilon}^i\gamma_5\lambda_i^a)(\bar{\lambda}^{bj}\gamma_5\lambda_j^c)\\
&+\alpha_{113}d^{abc}(\bar{\varepsilon}^i\gamma^\mu\gamma_5\lambda_i^a)(\bar{\lambda}^{bj}\gamma_\mu\gamma_5\lambda_j^c)+\alpha_{114}d^{abc}(\bar{\varepsilon}^i\sigma^{\mu\nu}\lambda_i^a)(\bar{\lambda}^{bj}\sigma_{\mu\nu}\lambda_j^c)\\
&+\alpha_{115_R}(T^a)_B^A(\bar{\varepsilon}^i\gamma_R\lambda_i^a)(\bar{\psi}_A\gamma_R\psi^B)\Big)
\end{aligned}
$$

$$\Delta^{(2)}= \tag{A.11}$$

$$
\begin{aligned}
&\int d^4x \Bigg(\beta_1(\bar{\varepsilon}^i\gamma^\mu\varepsilon_i)L^a c^a_\mu + \beta_2(\bar{\varepsilon}^i\varepsilon_i)\eta^{a\mu}(\partial_\mu A^a) + \beta_3(\bar{\varepsilon}^i\gamma^\mu\varepsilon_i)M^a(\partial_\mu A^a) \\
&+\beta_4(\bar{\varepsilon}^i\gamma_5\varepsilon_i)\eta^{a\mu}(\partial_\mu B^a) + \beta_5(\bar{\varepsilon}^i\gamma^\mu\varepsilon_i)N^a(\partial_\mu B^a) + \beta_6(\bar{\varepsilon}^i\gamma^\mu\varepsilon_i)\eta^a_\mu(\partial A^a) \\
&+\beta_7(\bar{\varepsilon}^i\gamma^\mu\varepsilon_i)\eta^{a\nu}(\partial_\mu A^a_\nu) + \beta_8(\bar{\varepsilon}^i\gamma^\mu\varepsilon_i)\eta^{a\nu}(\partial_\nu A^a_\mu) + \beta_9\epsilon^{\mu\nu\rho\sigma}(\bar{\varepsilon}^i\gamma_\mu\varepsilon_i)\eta^a_\nu(\partial_\rho A^a_\sigma) \\
&+\beta_{10}(\bar{\varepsilon}^i\varepsilon_i)M^a(\partial A^a) + \beta_{11}(\bar{\varepsilon}^i\gamma_5\varepsilon_i)N^a(\partial A^a) + \beta_{12}(\bar{\varepsilon}^i\gamma^\mu\varepsilon_i)U^*_{jA}(\partial_\mu A^{jA}) \\
&+\beta_{13}(\bar{\varepsilon}^i\gamma^\mu\varepsilon_i)U^{jA}(\partial_\mu A^*_{jA}) + \beta_{14}d^{abc}(\bar{\varepsilon}^i\gamma^\mu\varepsilon_i)A^aA^b\eta^c_\mu + \beta_{15}d^{abc}M^aA^bA^c \\
&+\beta_{16}d^{abc}(\bar{\varepsilon}^i\gamma_5\varepsilon_i)N^aA^bA^c + \beta^{abc}_{17}(\bar{\varepsilon}^i\gamma_5\varepsilon_i)M^aA^bB^c + \beta^{abc}_{18}(\bar{\varepsilon}^i\varepsilon_i)N^aA^bB^c \\
&+\beta^{abc}_{19}(\bar{\varepsilon}^i\varepsilon_i)\eta^{a\mu}A^b_\mu A^c + \beta^{abc}_{20}(\bar{\varepsilon}^i\gamma^\mu\varepsilon_i)M^aA^b_\mu A^c + \beta_{21}(T^a)^A_B(\bar{\varepsilon}^i\varepsilon_i)U^*_{jA}A^{jB}A^a \\
&+\beta_{22}(T^a)^A_B(\bar{\varepsilon}^i\varepsilon_i)A^aU^{jB}A^*_{jA} + \beta_{23}d^{abc}(\bar{\varepsilon}^i\gamma^\mu\varepsilon_i)B^aB^b\eta^a_\mu + \beta_{24}d^{abc}(\bar{\varepsilon}^i\varepsilon_i)M^aB^bB^c \\
&+\beta_{25}d^{abc}(\bar{\varepsilon}^i\gamma_5\varepsilon_i)N^aB^bB^c + \beta^{abc}_{26}(\bar{\varepsilon}^i\gamma_5\varepsilon_i)\eta^{a\mu}A^b_\mu B^c + \beta^{abc}_{27}(\bar{\varepsilon}^i\gamma^\mu\varepsilon_i)N^aA^b_\mu B^c \\
&+\beta_{28}(T^a)^A_B(\bar{\varepsilon}^i\gamma_5\varepsilon_i)B^aU^*_{jA}A^{jB} + \beta_{29}(T^a)^A_B(\bar{\varepsilon}^i\gamma_5\varepsilon_i)U^{jB}A^*_{jA}B^a \\
&+\beta_{30}d^{abc}(\bar{\varepsilon}^i\varepsilon_i)M^aA^{b\mu}A^c_\mu + \beta_{31}d^{abc}(\bar{\varepsilon}^i\gamma_5\varepsilon_i)N^aA^{b\mu}A^c_\mu + \beta_{32}d^{abc}(\bar{\varepsilon}^i\gamma^\mu\varepsilon_i)\eta^a_\mu A^{b\nu}A^c_\nu \\
&+\beta^{abc}_{33}(\bar{\varepsilon}^i\gamma^\mu\varepsilon_i)\eta^{a\nu}A^b_\mu A^c_\nu + \beta_{34}f^{abc}\epsilon^{\mu\nu\rho\sigma}(\bar{\varepsilon}^i\gamma_\mu\varepsilon_i)\eta^a_\nu A^b_\rho A^c_s + \beta_{35}(T^a)^A_B(\bar{\varepsilon}^i\gamma^\mu\varepsilon_i)A^a_\mu U^*_{jA}A^{jB} \\
&+\beta_{36}(T^a)^A_B(\bar{\varepsilon}^i\gamma^\mu\varepsilon_i)A^a_\mu U^{jB}A^*_{jA} + \beta_{37}(T^a)^A_B(\bar{\varepsilon}^i\varepsilon_i)M^aA^*_{jA}A^{jB} \\
&+\beta_{38}(\bar{\varepsilon}^i\gamma_5\varepsilon_i)(T^a)^A_BN^aA^*_{jA}A^{jB} + \beta_{39}(T^a)^A_B(\bar{\varepsilon}^i\gamma^\mu\varepsilon_i)\eta^a_\mu A^*_{jA}A^{jB} \\
&+\beta_{1_R}(\bar{\varepsilon}^i\gamma_R\varepsilon_j)(\bar{\Lambda}^{aj}\gamma_R\gamma^\mu\partial_\mu\lambda^a_i) + \beta_{2_R}(\bar{\varepsilon}^i\gamma_R\varepsilon_i)(\bar{\psi}_A\gamma_R\gamma^\mu\partial_\mu\psi^A) \\
&+\beta_{3_R}(\bar{\varepsilon}^i\gamma_R\varepsilon_i)(\partial_\mu\bar{\psi}_A\gamma^\mu\gamma_R\Psi^A) + \beta^{abc}_{4_R}(\bar{\varepsilon}^i\gamma_R\varepsilon_j)(\bar{\Lambda}^{aj}\gamma_R\lambda^b_i)A^c \\
&+\beta_{5_R}(T^a)^A_B(\bar{\varepsilon}^i\gamma_R\varepsilon_i)(\bar{\Psi}_A\gamma_R\psi^B)A^a + \beta_{6_R}(T^a)^A_B(\bar{\varepsilon}^i\gamma_R\varepsilon_i)(\bar{\psi}_A\gamma_R\Psi^B)A^a \\
&+\beta^{abc}_{7_R}(\bar{\varepsilon}^i\gamma_R\varepsilon_j)(\bar{\Lambda}^{aj}\gamma_R\gamma_5\lambda^b_i)B^a + \beta_{8_R}(T^a)^A_B(\bar{\varepsilon}^i\gamma_R\varepsilon_i)(\bar{\Psi}_A\gamma_R\gamma_5\psi^B)B^a \\
&+\beta_{9_R}(T^a)^A_B(\bar{\varepsilon}^i\gamma_R\varepsilon_i)(\bar{\psi}_A\gamma_R\gamma_5\Psi^B)B^a + \beta^{abc}_{10_R}(\bar{\varepsilon}^i\gamma_R\varepsilon_j)(\bar{\Lambda}^{aj}\gamma_R\gamma^\mu\lambda^b_i)A^c_\mu \\
&+\beta_{11_R}(T^a)^A_B(\bar{\varepsilon}^i\gamma_R\varepsilon_i)(\bar{\Psi}_A\gamma_R\gamma^\mu\psi^B)A^a_\mu + \beta_{12_R}(T^a)^A_B(\bar{\varepsilon}^i\gamma_R\varepsilon_i)(\bar{\psi}_A\gamma_R\gamma^\mu\Psi^B)A^a_\mu \\
&+\beta_{13_R}(T^a)^A_B(\bar{\varepsilon}^i\gamma_R\varepsilon_j)(\bar{\Psi}_A\gamma_R\lambda^a_i)A^{jB} + \beta_{14_R}(T^a)^A_B(\bar{\varepsilon}^i\gamma_R\varepsilon_j)(\bar{\lambda}^{aj}\gamma_R\Psi^B)A^*_{iA} \\
&+\beta_{15_R}(T^a)^A_B(\bar{\varepsilon}^i\gamma_R\varepsilon_j)(\bar{\psi}_A\gamma_R\Lambda^a_i)A^{jB} + \beta_{16_R}(T^a)^A_B(\bar{\varepsilon}^i\gamma_R\varepsilon_j)(\bar{\Lambda}^{aj}\gamma_R\psi^B)A^*_{iA}\Bigg)
\end{aligned}
$$

$$
\begin{aligned}
&\Delta^{(3)} = \qquad\qquad (A.12) \\
&\int d^4x \Bigg(\delta_1(\bar{\varepsilon}^i\varepsilon_i)(\bar{\varepsilon}^j\Lambda^a_j)M^a + \delta_2(\bar{\varepsilon}^i\gamma_5\varepsilon_i)(\bar{\varepsilon}^j\gamma_5\Lambda^a_j)M^a + \delta_3(\bar{\varepsilon}^i\varepsilon_i)(\bar{\varepsilon}^j\gamma_5\Lambda^a_j)N^a \\
&+\delta_4(\bar{\varepsilon}^i\gamma_5\varepsilon_i)(\bar{\varepsilon}^j\Lambda^a_j)N^a + \delta_5(\bar{\varepsilon}^i\varepsilon_i)(\bar{\varepsilon}^j\gamma^\mu\Lambda^a_j)\eta^a_\mu + \delta_6(\bar{\varepsilon}^i\gamma_5\varepsilon_i)(\bar{\varepsilon}^j\gamma^\mu\gamma_5\Lambda^a_j)\eta^a_\mu \\
&+\delta_7(\bar{\varepsilon}^i\gamma^\mu\varepsilon_i)(\bar{\varepsilon}^j\Lambda^a_j)\eta^a_\mu + \delta_8(\bar{\varepsilon}^i\varepsilon_i)(\bar{\varepsilon}^j\lambda^a_j)L^a + \delta_9(\bar{\varepsilon}^i\gamma_5\varepsilon_i)(\bar{\varepsilon}^j\gamma_5\lambda^a_j)L^a \\
&+\delta_{10}(\bar{\varepsilon}^i\varepsilon_i)(\bar{\varepsilon}^j\Psi_A)U^*_{jA} + \delta_{11}(\bar{\varepsilon}^i\gamma_5\varepsilon_i)(\bar{\varepsilon}^j\gamma_5\Psi^A)U^*_{jA} + \delta_{12}(\bar{\varepsilon}^i\varepsilon_i)(\bar{\Psi}_A\varepsilon_j)U^{jA} \\
&+\delta_{13}(\bar{\varepsilon}^i\gamma_5\varepsilon_i)(\bar{\Psi}_A\gamma_5\varepsilon_j)U^{jA}\Bigg)
\end{aligned}
$$

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